Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 2

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) **EITHER**

$115.5 = u_1 + (3-1)\times d \quad (115.5 = u_1 + 2d)$

$108 = u_1 + (8-1)\times d \quad (108 = u_1 + 7d)$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

1.5 (cups/day) ***A1***

Note: Answer must be written as a positive value to award ***A1***.

OR

$(d =) \ \dfrac{115.5-108}{5}$ ***M1A1***

Note: Award ***M1*** for attempting a calculation using the difference between term 3 and term 8; ***A1*** for a correct substitution.

$(d =) \ 1.5$ (cups/day) ***A1***

(ii) $(u_1 =) \ 118.5$ (cups) ***A1***

[4 marks]

(b) attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$0 = 118.5 + (n-1)\times(-1.5)$

$(n =) \ 80$ days ***A1***

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

(c) $(t_5 =) \ 625\times 1.064^{(5-1)}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution.

$\$ 801$ ***A1***

Note: The answer must be rounded to a whole number to award the final ***A1***.

[3 marks]

continued…

Question 1 continued

(d) (i) $(S_{10} =)$ (\$) 8390 (8394.39...) ***A1***

(ii) **EITHER**

the total cost (of dog food) ***R1***
for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

OR

the total cost (of dog food) ***R1***
from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

[3 marks]

(e) **EITHER**
According to the model, the cost of dog food per year will eventually be too high to keep a dog.
OR
The model does not necessarily consider changes in inflation rate.
OR
The model is appropriate as long as inflation increases at a similar rate.
OR
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.
OR
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either "model" is mentioned specifically, or other mathematical terms such as "increasing" or "discrete quantities" are seen. Do not accept a contextual argument in isolation, e.g. "The dog will eventually die".

[1 mark]
Total [13 marks]

2. (a) attempt to expand given expression **OR** attempt at product rule ***(M1)***

$$C = \frac{xk^2}{10} - \frac{3x^3}{1000}$$

$$\frac{\mathrm{d}C}{\mathrm{d}x} = \frac{k^2}{10} - \frac{9x^2}{1000}$$ ***M1A1***

Note: Award ***M1*** for power rule correctly applied to at least one term and ***A1*** for correct answer.

[3 marks]

(b) equating their $\frac{\mathrm{d}C}{\mathrm{d}x}$ to zero ***(M1)***

$$\frac{k^2}{10} - \frac{9x^2}{1000} = 0$$

$$x^2 = \frac{100k^2}{9}$$

$$x = \frac{10k}{3}$$ ***(A1)***

substituting their x back into given expression ***(M1)***

$$C_{\max} = \frac{10k}{30}\left(k^2 - \frac{300k^2}{900}\right)$$

$$C_{\max} = \frac{2k^3}{9}\left(0.222\ldots k^3\right)$$ ***A1***

[4 marks]

(c) (i) substituting 20 into given expression and equating to 426 ***M1***

$$426 = \frac{20}{10}\left(k^2 - \frac{3}{100}(20)^2\right)$$

$$k = 15$$ ***A1***

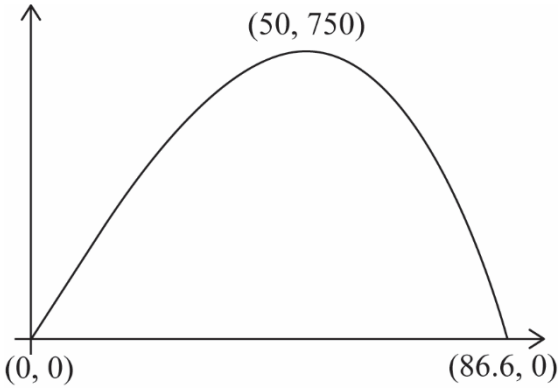
(ii) 50 ***A1***

[3 marks]

continued…

Question 2 continued

(d)



A1A1A1

Note: Award ***A1*** for graph indicating an increasing and then decreasing function (drawn in first quadrant), ***A1*** for maximum labelled and ***A1*** for graph drawn for positive x, passing through the origin and 86.6 which is marked on the x-axis or its coordinates are given.

[3 marks]

(e) setting their expression for C to zero **OR** choosing correct x-intercept on their graph of C ***(M1)***

$x_{\max} = 86.6 \quad (86.6025\ldots)$ litres ***A1***

[2 marks]
Total [15 marks]

3. (a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ ***(M1)***

$(4, 1)$ ***A1***

Note: Award ***A0*** if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula ***(M1)***

$\left(\frac{0-2}{6-2} = \right) -\frac{1}{2}$ ***(A1)***

therefore the gradient of perpendicular bisector is 2 ***(M1)***

so $y-1=2(x-4)$ $(y=2x-7)$ ***A1***

[4 marks]

(c) identifying the correct equations to use: ***(M1)***

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or finding points of intersection graphically ***(M1)***

$(3, -1)$ ***A1***

Note: Accept an answer expressed as "$x=3$, $y=-1$".

[3 marks]

(d) attempt to use distance formula ***(M1)***

$\mathrm{YZ} = \sqrt{(7-(-1))^2 + (7-3)^2}$

$= \sqrt{80}$ $\left(4\sqrt{5}\right)$ ***A1***

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule ***(M1)***

$\cos \mathrm{X\hat{Y}Z} = \frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $(=0.316227\ldots)$ ***(A1)***

Note: Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} = \right) 71.6°$ $(71.5650\ldots°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

continued…

Question 3 continued

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

Note: Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} =\right) 71.6°$ $(71.5650°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$= 24\ \mathrm{km}^2$ ***A1***

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

[1 mark]

Total [18 marks]

4. (a) attempt to use chain rule, including the differentiation of $\frac{1}{T}$ ***(M1)***

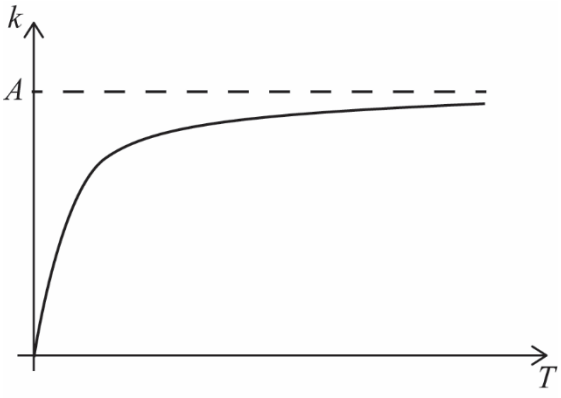
$\frac{\mathrm{d}k}{\mathrm{d}T} = A \times \frac{c}{T^2} \times \mathrm{e}^{-\frac{c}{T}}$ ***A1***

this is the product of positive quantities so must be positive ***R1***

Note: The ***R1*** may be awarded for correct argument from **their** derivative. ***R1*** is not possible if their derivative is not always positive.

[3 marks]

(b)



A1A1A1

Note: Award ***A1*** for an increasing graph, entirely in first quadrant, becoming concave down for larger values of T, ***A1*** for tending towards the origin and ***A1*** for asymptote labelled at $k = A$.

[3 marks]

(c) taking ln of both sides **OR** substituting $y = \ln x$ and $x = \frac{1}{T}$ ***(M1)***

$\ln k = \ln A - \frac{c}{T}$ **OR** $y = -cx + \ln A$ ***(A1)***

(i) so gradient is $-c$ ***A1***

(ii) y-intercept is $\ln A$ ***A1***

Note: The implied ***(M1)*** and ***(A1)*** can only be awarded if **both** correct answers are seen. Award zero if only one value is correct **and** no working is seen.

[4 marks]

continued…

Question 4 continued

(d) an attempt to convert data to $\frac{1}{T}$ and $\ln k$ ***(M1)***

e.g. at least one correct row in the following table

$\frac{1}{T}$	$\ln k$
$1.69491\ldots\times10^{-3}$	$-7.60090\ldots$
$1.66666\ldots\times10^{-3}$	$-7.41858\ldots$
$1.63934\ldots\times10^{-3}$	$-6.90775\ldots$
$1.61290\ldots\times10^{-3}$	$-6.57128\ldots$
$1.58730\ldots\times10^{-3}$	$-6.21460\ldots$
1.5625×10^{-3}	$-5.84304\ldots$
$1.53846\ldots\times10^{-3}$	$-5.62682\ldots$

line is $\ln k = -13400\times\frac{1}{T}+15.0 \quad \left(=-13383.1\ldots\times\frac{1}{T}+15.0107\ldots\right)$ ***A1***

[2 marks]

(e) (i) $c = 13400 \quad (13383.1\ldots)$ ***A1***

(ii) attempt to rearrange or solve graphically $\ln A = 15.0107\ldots$ ***(M1)***

$A = 3300000 \quad (3304258\ldots)$ ***A1***

Note: Accept an A value of $3269017\ldots$ from use of 3sf value.

[3 marks]

Total [15 marks]

5. (a) (i) 0.02 ***A1***

(ii) the probability of mutating from 'not normal state' to 'normal state' ***A1***

Note: The ***A1*** can only be awarded if it is clear that transformation is **from** the mutated state.

[2 marks]

(b) $\det\begin{pmatrix} 0.94-\lambda & 0.02 \\ 0.06 & 0.98-\lambda \end{pmatrix}=0$ ***(M1)***

Note: Award ***M1*** for an attempt to find eigenvalues. Any indication that $\det(\boldsymbol{M}-\lambda\boldsymbol{I})=0$ has been used is sufficient for the ***(M1)***.

$(0.94-\lambda)(0.98-\lambda)-0.0012=0$ **OR** $\lambda^2-1.92\lambda+0.92=0$ ***(A1)***

$\lambda=1,\ 0.92$ $\left(\begin{pmatrix}\frac{23}{25}\end{pmatrix}\right)$ ***A1***

[3 marks]

(c) $\begin{pmatrix} 0.94 & 0.02 \\ 0.06 & 0.98 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ **OR** $\begin{pmatrix} 0.94 & 0.02 \\ 0.06 & 0.98 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0.92\begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

Note: This ***M1*** can be awarded for attempting to find either eigenvector.

$0.02y-0.06x=0$ **OR** $0.02y+0.02x=0$

$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1A1***

Note: Accept any multiple of the given eigenvectors.

[3 marks]

(d) (i) $\begin{pmatrix} 0.94 & 0.02 \\ 0.06 & 0.98 \end{pmatrix}^5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.744 & 0.0852 \\ 0.256 & 0.915 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***(M1)***

Note: Condone omission of the initial state vector for the ***M1***.

0.744 $(0.744311\ldots)$ ***A1***

(ii) $\begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix}$ ***(A1)***

Note: Award ***A1*** for $\begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.25 & 0.25 \\ 0.75 & 0.75 \end{pmatrix}$ seen.

0.25 ***A1***

[4 marks]

Total [12 marks]

6. (a) (i) $\sqrt{10^2+8^2}$ ***(M1)***

$=12.8\ \ (12.8062\ldots, \sqrt{164})\ (\text{m s}^{-1})$ ***A1***

(ii) $\tan^{-1}\left(\frac{10}{8}\right)$ ***(M1)***

$=0.896$ **OR** $51.3\ \ (0.896055\ldots$ **OR** $51.3401\ldots°)$ ***A1***

Note: Accept 0.897 or 51.4 from use of $\arcsin\left(\frac{10}{12.8}\right)$.

[4 marks]

(b) $y=t(10-5t)$ ***(M1)***

Note: The ***M1*** might be implied by a correct graph or use of the correct equation.

METHOD 1 – graphical Method

sketch graph ***(M1)***

Note: The ***M1*** might be implied by correct graph or correct maximum (eg $t=1$).

max occurs when $y=5\ \text{m}$ ***A1***

METHOD 2 – calculus

differentiating and equating to zero ***(M1)***

$\frac{\text{d}y}{\text{d}t}=10-10t=0$

$t=1$

$y\ (=1(10-5))=5\ \text{m}$ ***A1***

METHOD 3 – symmetry

line of symmetry is $t=1$ ***(M1)***

$y\ (=1(10-5))=5\ \text{m}$ ***A1***

[3 marks]

(c) attempt to solve $t(10-5t)=0$ ***(M1)***

$t=2$ (or $t=0$) ***(A1)***

$x\ (=5+8\times 2)=21\ \text{m}$ ***A1***

Note: Do not award the final ***A1*** if $x=5$ is also seen.

[3 marks]

continued…

Question 6 continued

(d) **METHOD 1**

$t = \frac{x-5}{8}$ ***M1A1***

$y = \left(\frac{x-5}{8}\right)\left(10 - 5 \times \frac{x-5}{8}\right)$ ***A1***

METHOD 2

$y = k(x-5)(x-21)$ ***A1***

when $x = 13,\ y = 5$ so $k = \frac{5}{(13-5)(13-21)} = -\frac{5}{64}$ ***M1A1***

$\left(y = -\frac{5}{64}(x-5)(x-21)\right)$

METHOD 3

if $y = ax^2 + bx + c$

$0 = 25a + 5b + c$

$5 = 169a + 13b + c$

$0 = 441a + 21b + c$ ***M1A1***

solving simultaneously, $a = -\frac{5}{64},\ b = \frac{130}{64},\ c = -\frac{525}{64}$ ***A1***

$(y = -\frac{5}{64}x^2 + \frac{130}{64}x - \frac{525}{64})$

METHOD 4

use quadratic regression on $(5, 0),\ (13, 5), (21, 0)$ ***M1A1***

$y = -\frac{5}{64}x^2 + \frac{130}{64}x - \frac{525}{64}$ ***A1***

Note: Question asks for expression; condone omission of "$y =$ ".

[3 marks]

(e) trajectory of arrow is $y = x\tan 10 + 2$ ***(A1)***

intersecting $y = x\tan 10 + 2$ and their answer to (d) ***(M1)***

$(8.66, 3.53)\ ((8.65705..., 3.52647\ldots))$ ***A1***

$(15.1,\ 4.66)\qquad ((15.0859\ldots, 4.66006\ldots))$ ***A1***

[4 marks]

continued…

Question 6 continued

(f) when $x_{\text{target}} = 8.65705\ldots$, $t_{\text{target}} = \dfrac{8.65705\ldots - 5}{8} = 0.457132\ldots$ s ***(A1)***

attempt to find the distance from point of release to intersection ***(M1)***

$\sqrt{8.65705\ldots^2 + (3.52647\ldots - 2)^2}$ $(= 8.79060\ldots \text{ m})$

time for arrow to get there is $\dfrac{8.79060\ldots}{60} = 0.146510\ldots \text{s}$ ***(A1)***

so the arrow should be released when

$t = 0.311$ (s) $(0.310622\ldots$ (s)) ***A1***

[4 marks]
Total [21 marks]

7. (a) differentiating first equation. ***M1***

$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = \dfrac{\mathrm{d}y}{\mathrm{d}t}$

substituting in for $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ ***M1***

$= -2x - 3y = -2x - 3\dfrac{\mathrm{d}x}{\mathrm{d}t}$

therefore $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\dfrac{\mathrm{d}x}{\mathrm{d}t} + 2x = 0$ ***AG***

Note: The ***AG*** line must be seen to award the final ***M1*** mark.

[2 marks]

(b) the relevant matrix is $\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$ ***(M1)***

Note: $\begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}$ is also possible.

(this has characteristic equation) $-\lambda(-3 - \lambda) + 2 = 0$ ***(A1)***

$\lambda = -1, -2$ ***A1***

[3 marks]

continued…

Question 7 continued

(c) **EITHER**

the general solution is $x = Ae^{-t} + Be^{-2t}$ ***M1***

Note: Must have constants, but condone sign error for the ***M1***.

so $\frac{dx}{dt} = -Ae^{-t} - 2Be^{-2t}$ ***M1A1***

OR

attempt to find eigenvectors ***(M1)***

respective eigenvectors are $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (or any multiple)

$\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + Be^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ***(M1)A1***

THEN

the initial conditions become:

$0 = A + B$

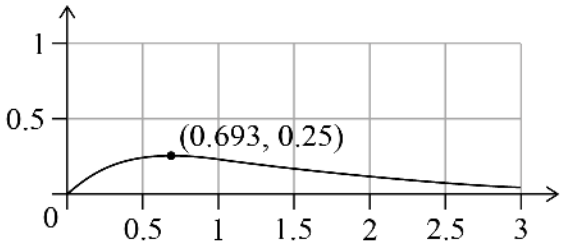
$1 = -A - 2B$ ***M1***

this is solved by $A = 1,\ B = -1$

so the solution is $x = e^{-t} - e^{-2t}$ ***A1***

[5 marks]

(d)



A1A1

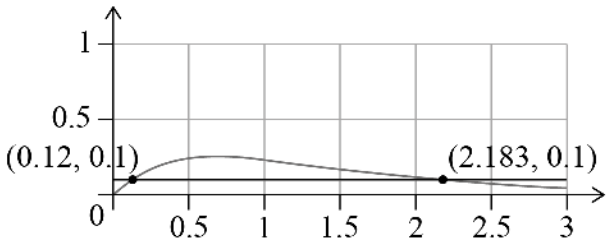
Note: Award ***A1*** for correct shape (needs to go through origin, have asymptote at $y = 0$ and a single maximum; condone $x < 0$). Award ***A1*** for correct coordinates of maximum.

[2 marks]

continued…

Question 7 continued

(e) intersecting graph with $y = 0.1$ ***(M1)***



so the time fishing is stopped between $2.1830\ldots$ and $0.11957\ldots$ ***(A1)***

$= 2.06(343\ldots)$ days ***A1***

[3 marks]

(f) *Any reasonable answer. For example:*
There are greater downsides to allowing fishing when the levels may be dangerous than preventing fishing when the levels are safe.
The concentration of mercury may not be uniform across the river due to natural variation / randomness.
The situation at the power plant might get worse.
Mercury levels are low in water but still may be high in fish. ***R1***

Note: Award ***R1*** for a reasonable answer that refers to this specific context (and not a generic response that could apply to *any* model).

[1 mark]

Total [16 marks]